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RAMON LULL AND THE THEORY OF VOTING[1]

1. Introduction

Ramon Lull's reputation among mathematicians and scientists has had less lustre than among students of language and literature. He does not feature in histories of Western mathematics. His obsessions with comparisons of objects in pairs and with the magnificent but impossible dream of the *General Art* led Donald Michie, an eminent computer scientist, to label Lull «one of the most inspired madmen who ever lived» (Gardner 1982, p. ix). Martin Gardner, the well-known mathematician and *Scientific American* columnist, wrote that Lull's life was «much more fascinating than his eccentric logic» (Gardner 1982, p. xiv). We aim to show that, in a branch of applied mathematics which has been permanently established only since 1951, Lull was inspired but not mad. He shared with Lewis Carroll (another of Gardner's favourite people) the unfortunate characteristic of being so far ahead of his time that nobody understood what he was talking about. His sole disciple was Nicolas of Cusa who transcribed the only known copy of Lull's most important paper, and who made an original contribution of his own, just over a century after Lull's death.

[1] The authors acknowledge with gratitude the support of the Leverhulme Trust for IMcL's work on the history of social choice; William Riker for an initial lead on Nicolas Cusanus; Alexander Murray for translating Cusanus and reviewing IMcL's translation of Lull's Latin text; and Robert Pring-Mill for reviewing JL's translation of Lull's Catalan text.

2. What is the theory of voting?

The deductive theory of voting is a branch of applied mathematics which explores the properties of different voting systems. It shows that voting procedures are often not so simple as is generally assumed, and that the subject is full of quirks and paradoxes. Perhaps that is why it has appealed to quirky and paradoxical scholars like Lull and Carroll.

It begins with majority rule. Majority rule between two candidates (or options) is a simple concept: it is intuitively obvious that it is «better» to accept the majority's preference (or judgment) rather than the minority's, at least so long as those in the majority are no worse qualified to form an opinion than those in the minority. Though this intuition was not mathematically formalised until 1952 (May 1952), it was perfectly obvious to the inventors of democracy, the Greeks of the 5th century BC.

But if there are more than two candidates or options, what does majority rule mean? Greek democrats seem never to have faced this question. Votes in the Athenian Assembly were always on binary propositions (in other words, motions to take some course of action where the alternatives were just Yes and No). Elections to office, where the problem of multiple candidates would have come up, were rare because Greek democrats had a strong preference for choice by lot over choice by election: evidently they thought the former was more democratic. (See e.g. Rodewald 1975). The first known discussion of a problem of majority rule with more than two candidates comes in a letter of Pliny the Younger, c. 105 AD (Radice 1969, pp. 220-4; discussed by Farquharson 1969, *passim,* and by Riker 1986, pp. 77-87). Pliny realised that in a vote among three courses of action, the outcome if a three-way vote was taken might differ from the outcome by conventional committee procedure, in which one option is paired against another and the survivor pitted against the third. He tried to manipulate proceedings by holding a threeway vote instead of two binary votes: but his manoeuvre was matched by his main opponent, so that the outcome was the same as it would have been without manipulation. This is the first known discussion of what are now called «sincere» and «sophisticated» (or «strategic») voting.

At this point we must break our chronological narrative in order to explain the later work on the paradoxes of majority rule which Lull and Cusanus partly anticipated. In 1770 J.-C. de Borda gave a paper to the Royal Academy of Sciences in Paris in which he pointed out that the «ordinary method of elections» was defective because it could fail to choose the true majority winner. The «ordinary method» was what is now called «plurality» or «first-past-the-post»; it was the method which

Pliny had manipulatively tried to introduce, and which is in use in public elections in Britain, the USA, and a number of other former British colonies. In a plurality election, each voter indicates only his/her first preference; the candidate with the largest number of first preferences is elected, whether or not that number is greater than half of the total votes cast. Borda pointed out that it was quite possible for a candidate whom more than half of the voters thought the worst to be elected under the plurality rule. By implication, his criterion for a good system was that it should select that candidate whom a majority of the voters preferred to each of the others. His recommended procedure was the now familiar rank-order count: each voter gives 1 point to his least-liked candidate, 2 to the next least-liked, and so on up to n, where there are n candidates, to his favourite. In technical literature this procedure is called the «Borda count» in honour of its (supposed) inventor. Borda's paper was published in 1784.

The following year, the Marquis de Condorcet published his enormous *Essai sur l'application de l'analyse à la probabilité des décisions rendues à la pluralité des voix,* the Old Testament of the theory of voting. Condorcet made two discoveries relevant to Borda: first, that Borda's procedure sometimes failed to select the majority winner; second, that the majority winner might not even exist. The first discovery was important because the Condorcet criterion («Compare each candidate against each of the others. Select the one who wins a majority against every other») is the natural, and best, extension of the idea of majority rule to the case of more than two candidates. Borda had himself appealed to it without defining it. The latter alarming discovery is most easily shown in the simplest case (not the one Condorcet himself used). There are three voters, A, B, and C, and three candidates x, y, and z. A prefers x to y and y to z. B prefers y to z and z to x. C prefers z to x and x to y. If they vote on the candidates two at a time, this three-person society will prefer x to y by two votes to one, y to z by two votes to one - and z to x by two votes to one! Whichever candidate the society chooses, there exists another who would beat him/her by a simple majority vote. This situation is called, in a term introduced by C.L. Dodgson (Lewis Carroll) in 1876, a «cycle». Condorcet's solution to the problem of finding the majority candidate, should one exist, is as noted to conduct exhaustive pairwise comparison among all the candidates. For three candidates, this involves three comparisons; for four candidates, six; for nine candidates, 36; for n candidates, $\{n\ (n - 1)/2\}$. The winner is then the candidate who wins all his/her comparisons. Condorcet's successive solutions to the cyclical case, as he wrestled with the problem for the rest of his life, are unclear and obscure, although it has been very recently argued (especially

by Young 1988) that Condorcet does have a coherent solution. Fortunately, exploration of this difficult issue is beyond the scope of this paper.

Condorcet's work was forgotten by around 1820. His and Borda's discoveries were repeated from scratch by Dodgson, who was again utterly neglected. The New Testament was not written until the 1950s, in the form of Arrow (1951) and Black (1958). These works reinstated the theory of voting, as part now of an academic discipline called «social choice»; and pushed the problem of cycles to new depths of paradox and impossibility. (The best reviews, for readers who would like to learn more about social choice, are Black (1958) for Borda, Condorcet, and Dodgson, and Riker (1982) for the 20th-century rediscovery). In what follows we aim to show that Arrow and Black's inauguration of voting theory was not, as has been generally believed, the third, but the fourth.

3. Lull's contribution

By Lull's time, the problem of elections had resurfaced in the Church, and above all in monastic orders, which had to conduct their own elections independently of the hierarchy of the Church. The Rules of the Dominican Order, for instance, which were drafted early in the 13th century, contain elaborate directions for elections to abbacies and other higher posts in the Order (Galbraith 1925, esp. pp. 5, 33, 46, 64, 103, 114, 226-36). The favourite phrase among churchmen was that elections should be decided by the *maior vel* [or *et*] *sanior pars*. But this formula was deeply unsatisfactory, as every defeated minority could, and usually did, immediately claim that it was the *sanior pars,* and disputed elections were legion. They culminated in the Great Schism in the papacy between 1378 and 1417, when there were two and at one point three claimants to the papacy. Lull made the most constructive contribution to this debate, which was also one of the first systematic contributions to the deductive theory of voting.

His theory of voting appears in two places that we have found: in *Blanquerna* (c.1282-1287), and a short paper entitled *De arte eleccionis* (Honecker 1937a, pp. 308-9) written in 1299.

Lull's voting procedure is slipped in to *Blanquerna* at chapter 24, where as elsewhere in the novel the story-line is suspended for a piece of practical advice. The Abbess has died, and the nuns are deciding what to do.

> All the sisters wanted to elect their abbess by their usual electoral method, but Natana said that she had heard of a new electo-

ral method, which consisted in art and figures; this art follows the conditions laid out in *The Book of the Gentile and the Three Wise Men,* which follows *The Art of Finding Truth.* «By this method», said Natana, «truth is found, and by this truth we will be able to find which of us is best and most suitable to be our abbess».

All the sisters asked Natana to reveal the way in which, through art, they could find and elect the sister best suited to be abbess. This was Natana's reply: «I will briefly tell you about the principles of the art of election. This art is divided into two parts: the first part involves electing those who will elect the leader; the second part concerns the way in which they should elect the superior. So I will first tell you about the first part and then the second».

Natana said: «There are twenty of us in this chapter who have the right to vote in the election of our leader. According to the art we must elect from these twenty sisters an odd number, which should be five or seven, because this number is more appropriate for an election than any other; and the number seven is more appropriate than the number five. All the sisters should first take an oath to tell the truth. Then the first sister should be asked in secret which of the nineteen are most suited to be among the seven who can elect the superior. Afterwards, the second sister should be asked, then the third, and so on, in order, until the last. And on each occasion the answer of each sister should be written down. In the end, let it be ascertained which of the sisters have won the most votes, and those who have the most votes will be the seven sisters to elect the abbess.

»The second part of the election concerns how the seven electors elect their leader. Firstly, the seven electors should agree upon a certain number and upon certain names for election, as they best see fit. They should compare them with each other according to four conditions, namely: which of them best loves and knows God, which of them best loves and knows the virtues, which of them knows and hates most strongly the vices, and which is the most suitable person.

»Each of the seven electors can choose one person to be in the total number of those from whom the superior will be elected and each elector shall herself be among that number. So that you can understand the art more clearly, let us suppose that the given number of people from which our superior is to be elected is nine. Thus, the seven should be divided into two groups: two in one

and five in the other. The five should decide which of the two should be elected, and write in secret the name of the one who has won more votes. Afterwards, the sister who has won more votes should be compared with another of the five; this other sister should replace the one who has been defeated by reason of fewer votes. The defeated sister should be put in the place of the sister who is compared with the first or the second. This procedure should be repeated, in order, with all the others, and the eighth and ninth candidates, who are not among the electors, should be included in this number. Therefore, taking this number as an example, thirty-six compartments[2] will be produced in which the votes of each candidate will appear. The candidate to be elected should be the one with the most votes in the most compartments».

When Natana had explained the art of election, one of the sisters asked her: «If it turns out that some candidates have as many votes as each other in the compartments, what procedure does the art recommend?» Natana replied: «The art recommends that these two or three or more should be judged according to art alone. It should be found out which of these best meets the four aforementioned conditions, for she will be the one who is worthy to be elected».

All the sisters were very pleased with the art and the electoral system. They all said that if they followed this art there could be no error in the election. So they all established a rule according to which they would always carry out their elections according to the way explained by Natana, and they began to find out about the art and learn it. After a few days, they had an election according to the art and discovered that Natana was to be abbess.

Natana was elected abbess. She was very upset by being honoured in this way. She blessed God for honouring her above all the other sisters. However, she thought that the sisters might have made a mistake in the art and wanted to see the thirty-six compartments in which the art was arranged, to see if they had made a mistake and that she should not, in fact, be abbess; in which case, they should elect the sister indicated by the working of the art. Natana and the other sisters who had not been among the seven electors checked the method which had been followed according to the art in the election and confirmed that the art had been followed as was indicated. Then Natana began to think

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[2] The Catalan word is 'cambres'. We assume that Lull had in mind the sort of triangular half-matrix he later draws in *De Arte Eleccionis*.

> deeply about how she would learn and be able to rule herself and the sisters, and every day she meditated as to how she could manage the convent in right ways (*Blanquerna* ch. 24).

The theory of elections recurs in later chapters, as when Blanquerna is elected abbot «according to the manner of election whereby Natana had been elected abbess». Later still, he is proposed for a bishopric. He does not want it, because it would mean giving up the contemplative life. Most of the electors nevertheless vote for him on the advice of the retiring bishop, but his enemy the Archdeacon leads a faction who do not want him because he might forcibly turn them from secular to regular clergy. The Archdeacon «opposed the holding of an election according to the art». One takes place «without the art», but it leads to a dispute, the majority electing Blanquerna and the minority the Archdeacon. Both sides go to Rome, where the pope rules in favour of the reluctant Blanquerna. Thus people who oppose the correct art of elections come to a suitably sticky end (*Blanquerna* chs 24, 60, and 67).

There are several striking features about Lull's method. Firstly, it is a two-stage method. Like Condorcet and the American Federalists (Madison, Hamilton and Jay) five centuries later, Lull seems to wish to compromise between democracy and giving a more decisive voice to better qualified electors. This may represent Lull's attempt to compromise between the rival principles of *maior pars* and *sanior pars*. Secondly, the election is to be made on multiple (four) criteria. It is not clear from the text, but is at least possible, that Lull realised that multiple-criterion decision-making may lead to difficulties in aggregating from individual to social orderings (for which see e.g. Arrow and Raynaud 1986). Third and most important, it is a method of exhaustive pairwise comparisons. The 36 *cambres* («compartments» or «cells») represent the 36 combinations of two candidates from nine — as it would now be written $n(n-1)/2$ for $n = 9$. Lull was one of the first mathematicians in the West to explore the combinations and permutations of smaller numbers from greater ones. In particular, the principle of selecting pairs of people (or properties) from longer lists appears all over the place in the General Art. Gardner (1982, p. 18) truly remarks that «Lull's mistake ... was to suppose that his combinatorial method had useful applications to subject matters where today we see clearly that it does not apply». However, the application to voting rules is perhaps Lull's most fruitful use of the principle of pairwise combination. Unlike others it is an entirely appropriate application of the mathematics of combinations, not repeated until 1785.

Lull advocates the choice, not of the Condorcet winner but of the candidate who wins most votes when all the pairwise comparisons are added together. That is the natural interpretation of the sentence translated above as «The candidate to be elected should be the one with the most votes in the most compartments». This is in fact a «Borda», not a «Condorcet» procedure. It is identical to the second method proposed by Borda in his paper of 1770. As both Borda and Black (1958, p. 158) showed, Borda's second method must always lead to the same result as his first, the rank-order count. It is exactly the same as running a Borda count with a score of 0 for bottom place, 1 for second-bottom, and so on up to (n - 1) for top. Thus Lull's first procedure, like Borda's second, appears to be a «Condorcet» procedure, because it make comparisons for every pair of the candidates. But instead of simply selecting the candidate who wins all eight of her comparisons, it tots up the votes she gets in each comparison. This is actually a rank-order procedure.[3]

The electoral procedure in *De Arte Eleccionis* was devised, Lull tells us, at Paris on July 1, 1299. He begins with a triangular half-matrix showing the 36 pairwise comparisons of nine candidates labelled from «b» to «k» consecutively:

bc	cd	de	ef	fg	gh	hi	ik
bd	ce	df	eg	fh	gi	hk	
be	cf	dg	eh	fi	gk		
bf	cg	dh	ei	fk			
bg	ch	di	ek				
bh	ci	dk					
bi	ck						
bk							

[3] There is another possible interpretation of the phrase «The candidate to be elected should be the one with the most votes in the most compartments». Lull may have meant «the candidate who wins the largest number of her contests». This is the Condorcet winner if there is one candidate who wins all eight. If there is a cycle, there may be three or more candidates who each win seven and lose one. If there was any other evidence from the text that Lull had this idea in mind, then he would not only have discovered cycles 500 years before Condorcet but also discovered what is now called the Copeland method 700 years before Copeland. But alas we believe that such an interpretation puts too much strain on the text and we settle for the obvious one.

The full text runs:

The art of elections

In the Holy Church, good elections are greatly needed to choose representatives, as the church is to be governed by them and they must fight her enemies –sinners, infidels, and schismatics. These elected representatives cannot do this unless they are good men, well adapted to the service of their Mother, who is good and noble, namely the Holy Catholic Church. For the Church suffers greatly from those who pretend to be her faithful sons, but who are in fact evil men who do harm to their Mother and usurp her possessions.

Therefore we wish to propose a method of electing a representative in accordance with the Third Figure of the General System of Knowledge, such that by following the stages of this method the electors may publicly choose the better person and that, if they do not choose the best, it will be obvious to everyone in the chapter that they are choosing the worse candidate and perjuring themselves without any colour of an excuse. The method of election is as follows.

Let *b* stand for the first candidate to have entered the Church,[4] *c* the second candidate and so on to *k*, so that the first person to have held church office is called *b*, the second *c* and so on. If there are more than nine candidates in the church, the cells in the figure above are multiplied by adding *l;* if there are eleven, then *m*. If there are more candidates than letters in the alphabet, we use numbers, so that one brother is called candidate 1, another 2, and so on in the same way.

First, let all voters take an oath that they will elect the better and more suitable candidate. Next, let the electors sit down, and let *b* and *c* stay standing at the side, near enough to hear what the electors say, and so that they themselves are visible to all. Then

[4] Latin «quae in ecclesia prius fuit recepta». This could alternatively mean «the first person to have entered the church» i.e. the first to step into the building, in which case «the first person to have held church office» should rather be translated «the first person to enter the room». The distinction between the two meanings has non-trivial consequences. The first person to enter a set of pairwise votes is always at a disadvantage, if voters vote «sincerely» in accordance with their preferences. So if the interpretation in our translation is correct, Lull's proposal biases selection against the most senior. If the interpretation in this note is correct, it biases against the most punctual, or the most pushy. Unfortunately we cannot decide from the Latin which is correct.

let *d* ask everyone who is sitting down which of *b* and *c* he prefers for abbot, prior, or bishop, and go on until he has asked them all. The votes of the several electors are then counted and if *b* has more than *c* let *c* sit down in his place and *b* remain standing; or *vice versa*. Then let *d* be set against *b:* let *c* stand up and ask everybody which of *b* and *d* he prefers as prelate, and if *b* has more votes, let *d* sit down. Let *e* go with *b* and let *c* or *d* or another ask each voter in the presence of all, which of *b* and *e* he prefers for master. Suppose *e* beats *b,* then let *f* go to *e* or *vice versa;* if *b* is defeated, let *c* ask each which of *e* and *f* he prefers for master; and so on in order to *k,* so that with *k* the election comes to an end whether *k* wins or loses. Likewise if there are ten in the chapel, the last decision comes with *l* in order, as just explained.

This method of election is most useful and safe, because it does not involve secret ballots and private pacts, which are more open to fraud than the above method. Those who choose openly are so placed as to be in disgrace with their colleagues if they choose badly. Those who elect in secret are not.

This new method of election is also good in that it is more general than any other method can be, because for every candidate in the chapter a reckoning is made of each elector's wishes in one of the cells of the above figure, and so each elector is happier with the result.

Similarly, by this method of election any person in the chapter might suggest that it would be a good and honourable thing if this method were to be used in electing prelates, and he would obtain friends in the church, bring about peace, and avoid animosity, so that he might be chosen when the election came, and his colleagues would show mutual love, so that in an election one should stand for another and so the standing of the chapter would be raised through the charity, justice, prudence, and other virtues shown by the brethren towards each other.

If an election of persons not present is to be made, it should be done by the aforesaid method.

This method of election was devised in Paris in the year of the incarnation of our Lord Jesus Christ 1299 on the first of July. Thanks be to God.

This procedure, unlike that described in *Blanquerna,* is a Condorcet procedure. Its modern counterpart is the successive voting rule used in, for instance, the Norwegian parliament (Rasch 1987). It uses the matrix

notation previously thought to have been invented by C.L. Dodgson («Lewis Carroll») nearly six centuries later (Dodgson 1876). Because the winning candidate must have beaten at least one other, it cannot select a Condorcet loser. Thus it is clearly better than the plurality rule used in British and American public elections, which, as Borda was the first to show explicitly, can select a person whom a majority of the voters rank last. Such an absolute majority loser is the worst kind of Condorcet loser. If a Condorcet winner exists, Lull's scheme will select him/her. However, it cannot detect the existence of cycles because not every comparison in the matrix is actually used in selecting the winner.

4. Nicolas Cusanus

Lull appears to have been too far ahead of his time to have made any impact. In his *Vita coetana* he complained that nobody understood him when he lectured in Paris because of his «Arabic way of speaking» (Bonner 1985, I, pp. 29, 38). But one person who read *De Arte Eleccionis* may have been the transcriber of the only known copy: Nicolas Cusanus. Cusanus studied first at Heidelberg, then at Padua, where he gained his doctorate in 1423, then at Cologne. Padua was one of the leading intellectual centres of Europe, and Lull's mathematical, as well as his theological, works were on the curriculum there (Sigmund 1963, pp. 22-35). Cusanus was active in the conciliar movement of his time. The Council of Constance (1414-17) was convoked to try to end the papal schism; it succeeded in ousting all three of the current contenders for the title of pope, and electing one of its choice. Its voting procedures were contentious, and included a weighted-voting scheme (voting by «nations») to ensure that the Italian electors did not carry the day by sheer force of numbers. (Most council members were bishops and Italy had the largest number of bishoprics). Thus questions of voting procedure were part of the political agenda of the day.

Cusanus' main work of political theory, *De Concordantia Catholica,* was written while he was attending the Council of Basel (1431-4). It defends the rights of councils to elect popes; and it discusses voting procedures in chapters 36 and 37 of Book III. These chapters deal with the election of a Holy Roman Emperor rather than a pope. Cusanus first discusses the need to prevent «practicas absurdissimas et inhonestissimas» («the most absurd and dishonest practices») and notes that because particular electors come from particular «towns and camps of the empire», «turpiter foedatae electiones per iniustas pactiones fieri dicuntur» («elec-

tions could be said to be disgracefully rigged by unjust pacts»). He then describes his procedure. We quote chapter 37 in full.

> **535** When the electors of the Holy Roman Empire wish to proceed to the election of a future emperor let them convene on the day arranged. In all humility and with the utmost devotion to the things of God, let them strip themselves of all sin so that Christ should be as Lord in their midst, and the grace of the Holy Spirit whom they have invoked.
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> After a devout entry to the proceedings let them establish the names of all those persons conceived as worthy, by dint of both external and internal qualities, to function in so majestic an office. So that the election may be made with unrestricted liberty and secrecy, and free of all fear, let the electors then proceed as follows. After first swearing an oath on the Lord's altar to elect the person whom their free conscience shall duly judge best, the electors should get a notary to write, on slips of paper precisely equal in size, the names of the candidates. One name should go on one slip of paper, and after the name a clear digit - 1, 2, 3 and so on - until there is a slip for each of the persons whom their previous deliberations have agreed to be worthy of consideration.
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> **536** Let us now suppose that ten persons, found from throughout Germany, have been thus deemed worthy, and that it is from among these that the most worthy is to be elected by common resolve. One, only, of these names is to be written on each slip. Under or beside the name a number is to be written (from one to ten). Ten slips, each with the name of one of the ten candidates, should then be given to each elector. When the electors have got their slips each of them should go off alone, secretly (with a secretary if he cannot read), and, putting all ten slips in front of him, read the name on each.
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> **537** Let the elector then ponder in his conscience, in God's name, which of all the candidates is least suitable. Let him make in ink a single stroke on the appropriate slip to indicate the number «one». Then let him consider who is the next-least worthy, and write with two simple strokes of the pen the number «two» [in Roman characters - translators]. So he should go on, through the others, until he comes to the candidate who is in his judgment the best. On his slip he will write the number ten —or whatever number corresponds to the total of candidates.

538 It is recommended that all electors write with the same ink and with similar pens and similar strokes, long or short, as they shall agree. In this way no-one's writing will stand out as recognisable from the others', and the electors will therefore be able to act with greater freedom, and general harmony be preserved among them.

539 When the marks have been thus made let each elector take his own bunch of slips in his hand and throw them, with his own hand, into an empty bag hung up in the midst of the electors. When all the slips are in the bag the priest who celebrated Mass should be summoned, and a teller with a writing-block on which the names of those to be elected —ten in the example— are written in the established order. Sitting in the middle of the electors the priest should then take the slips out of the bag one by one, in whatever order his hand may find them, and read out the name and the number written on it. Meanwhile the teller at his side should note down each number as it comes. When they are all recorded the teller must add up the numbers by each name, and the candidate who has collected the highest total will be emperor.

540 By this method innumerable malpractices can be avoided and indeed no malpractice is possible. In fact no method of election can be conceived which is more holy, just, honest, or free. For by this procedure no other outcome is possible, if the electors act according to conscience, than the choice of that candidate adjudged best by the collective judgment of all present. Nor will any surer method be discovered for reaching so infallible a formulation of collective decision. For this method takes account of all comparisons of candidate to candidate - in whatever groupings or combinations - that any elector can make (*quoniam omnes comparationes omnium personarum et omnes mixturae et syllogismi per unumquemque ex electoribus factibiles in hoc modo includuntur*). I have myself been unable to find a better method than this even after much effort; and you can safely take it that a more perfect method cannot be found. (Kallen 1964, pp. 448-50, translated by A. Murray).

What is the relationship between Lull's ideas and Cusanus'? We must first establish how much of Lull's work Cusanus knew. *De Arte Eleccio-*

nis was, and still is, in his library; he may have copied it out himself. He knew of the existence of *Blanquerna*, as it appears on a Latin handlist of Lull's works in Cusanus' library; but he probably had not read it (Honecker 1937b, pp. 570-1).

Thus the scheme of Lull's that Cusanus certainly knew was a «Condorcet» scheme of public voting; and Cusanus proposed instead a «Borda» scheme with secret voting. Was this simply because he failed to understand the merits of Lull's scheme? (cf. Kallen 1964, p. 448, fn. to ch. 37) We think not. Cusanus' own words quoted above, together with the high intellectual regard for Lull reflected by his collecting, and probably transcribing, *De Arte Eleccionis*, suggest that he considered Lull's scheme and rejected it. Neither writer gives a mathematical or logical justification for his scheme: such justifications had to await Condorcet and Borda. But there are arguments in favour of both the Condorcet and the Borda principles,[5] and the debate between them is open to this day. We infer that Cusanus rejected Lull's scheme on principle and not out of misunderstanding.

The issue between secret and open voting is also still open. Both writers wish to eliminate strategic voting, but they make opposite recommendations. This is because they are discussing different situations. Lull is concerned with the members of a cathedral or abbey chapter voting to select their own leader. In this case, the electors are all known to each other, and must continue to live together after the vote. Thus it is reasonable to demand open voting: a voter will then be constrained by his fellow-voters' knowledge of his preferences. In general, this is the argument for open voting in committees where the members must trust one another if business is to be done. Cusanus is concerned with a body of electors meeting once only and suspicious of one another's strategic voting intentions before the election starts. He had first-hand experience of this at the Council of Basel, and it was a well-remarked feature of the conciliar movement. In this case, increasing the amount of information about others' votes available to each voter *increases* the opportunities and incentives for strategic voting of a log-rolling kind. (Log-rolling is vote-trading: «You vote for my pet scheme, or candidate, and I will vote for yours».) It was presumably in part to prevent this that the Councils voted by nations.

In conclusion, then, we find that Lull and Cusanus, both hitherto

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[5] The Condorcet principle is, as already noted, «Compare each candidate against each of the others. Select the one who wins a majority against every other». The Borda principle is «Select the candidate who is on average highest on the voters' ballot papers». If each voter ranks the candidates in order of preference, both the Borda winner and the Condorcet winner (if the latter exists) can be easily calculated.

unknown to historians of social choice, anticipated the work of Condorcet, Borda and Dodgson by over 500 years. Although neither gives full reasons for adopting his favourite scheme(s), both clearly understand some of the logical issues involved. Much remains tantalisingly unknown. Lull was deeply learned in Arab thought, and he no doubt got some of his mathematical ideas from this source. Should we look to the Arab world for the origins of social choice as well as of algebra?

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RESUM

Les modernes teories matemàtiques de la votació desenvolupades en la dècada dels 50, es pensava que tenien tres predecessors: Borda i Condorcet al S. XVIII a França, i Dodgson (Lewis Carroll) al S. XIX a Anglaterra. Aquest article mostra que tenien dos predecessors: Ramon Llull, que en *Blanquerna* i l'*Ars electionis* va desenvolupar dues variants del sistema de Condorcet; i Nicolau de Cusa, la biblioteca del qual contenia una còpia de l'*Ars electionis* de Llull, i que en el seu *De concordantia catholica* va desenvolupar una tècnica de Borda. La notació matricial emprada en el projecte lul·lià (i tan freqüent en la seva Art), es pensava que havia estat aplicada a la teoria de la votació per primera vegada per Dodgson uns sis segles més tard.